INDIAN STATISTICAL INSTITUTE
Probability Theory I: B. Math (Hons.) I
## Semester I, Academic Year 2018-19
Midsem Exam

Date: 11/09/2018   Total Marks: 30   Duration: 10 am - 12 pm

- **Show all your work and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.**

- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.**

1. Consider the following schematic diagram of a drainage network model (as described in the class), where each of Paths 1 - 5 (as shown in the figure below) behave <u>independently</u> of each other. Suppose that each path is open with probability $p \in (0, 1)$.
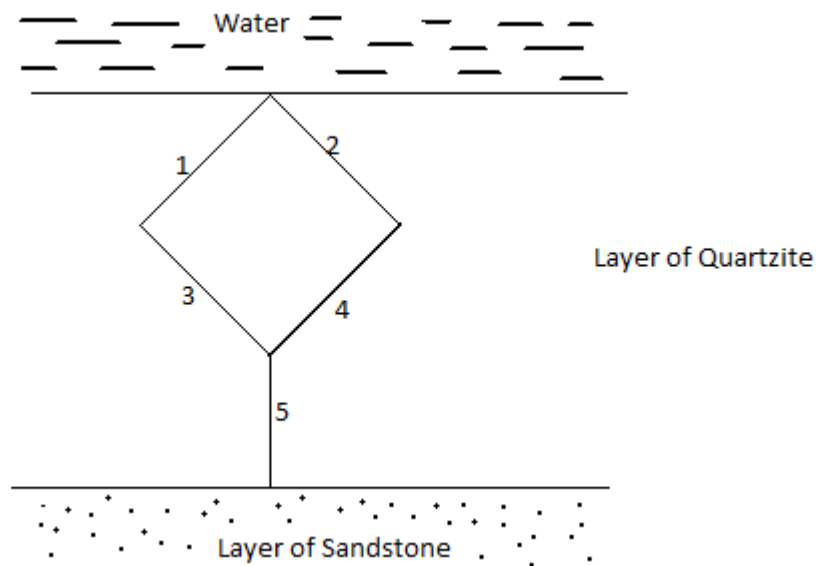
   (a) (5 marks) If it is given that exactly four paths are open, calculate the probability that water can pass through the layer of quartzite to the layer of sandstone.
   (b) (5 marks) If it is given that water can pass through the layer of quartzite to the layer of sandstone, calculate the probability that exactly four paths are open.

Please turn over to the other side.

2. (10 marks) Consider Polya's Urn Scheme as described in the class (starting with b black balls and r red balls, and adding c balls at each stage). Define, as in the class, R_i to be the event that the ith draw yields a red ball. Calculate $P(R_2|R_3)$.

3. (10 marks) Let N be the number of empty poles when r flags of different colours are displayed randomly on n poles arranged in a row (here $r, n \in \mathbb{N}$ with $r \geq n$). Assuming that there is no limitation on the number of flags on each pole, compute $P(N = 0)$.